# Module-2
## Elementary Concept of Magnetic Materials

* A magnetic is a material or object that produces a magnetic field.

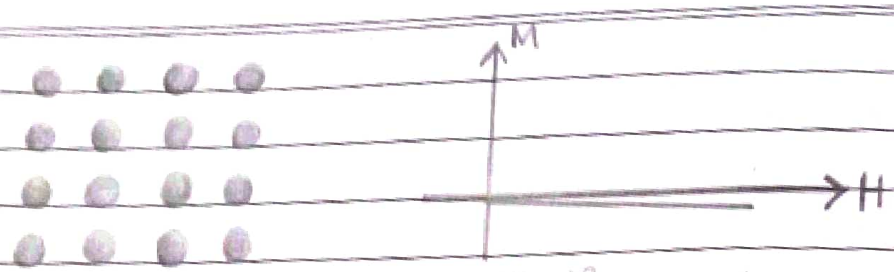
eg: Iron, steel, nickel, cobalt

* Classified into 3:

- Diamagnetic material
- Paramagnetic material
- Ferromagnetic material

* Diamagnetic materials

- Diamagnetic substances are composed of atoms which have no net magnetic moments.
- i.e. all the orbital shells are filled and there are no unpaired electrons.
- when exposed to a field, a negative magnetization is produced and thus the susceptibility is negative.
- Susceptibility: measure of how much a material will become magnetized in an applied magnetic field.




Magnetization and magnetic field intensity.

## * Paramagnetic materials

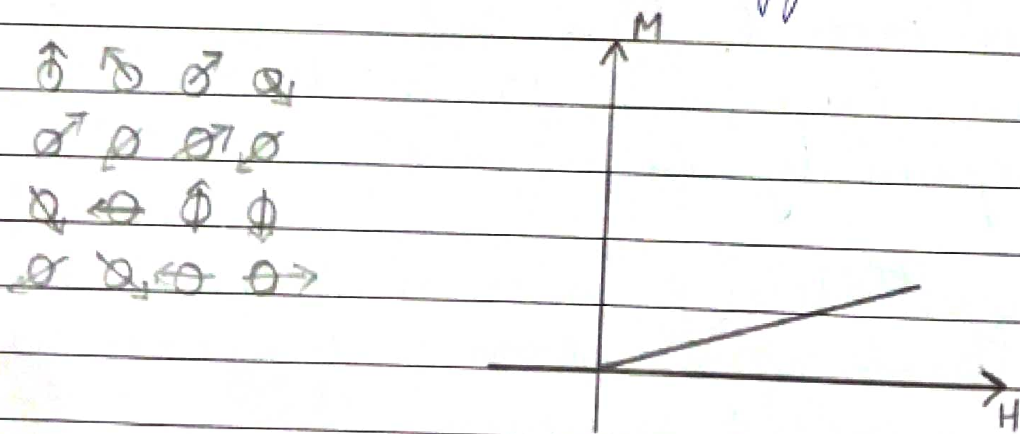
Some of the atoms or ions in the materials have a net magnetic moment due to unpaired electrons.

When placed in a magnetic field, magnetic field within the material gets enhanced.

When placed in a non uniform MF, it tends to move from low to high field region.

Have permanent dipole moment.

eg: Aluminium, Sodium, Copper choride.



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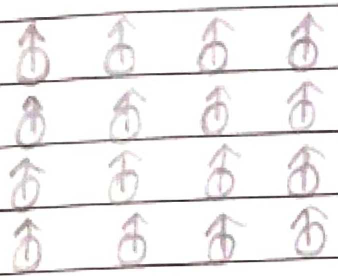
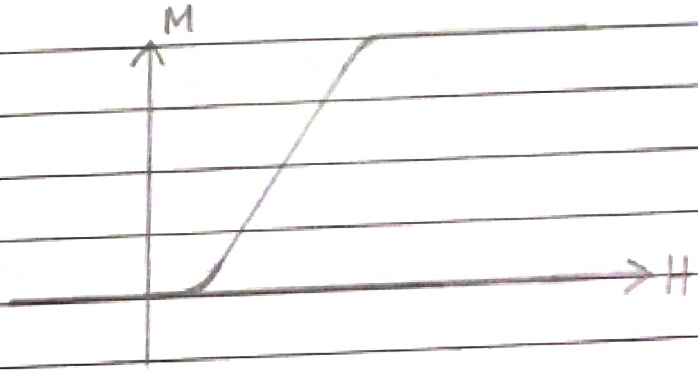
## ★ Ferromagnetic Materials

- Gets magnetized even by weak magnetic field.
- Ferromagnetic materials exhibit parallel alignment of moments resulting in large net magnetization even in the absence of a magnetic field.

eg: Fe, Ni, Co.






## Classification of magnets

### ★ Permanent magnet

- Retain magnetism even after removing the external magnetic field.
- Are retain their magnetism and magnetic properties for a longer time. Strongly magnetized hard materials make up permanent magnets.
- Hardened steel and alloy of steel can be transformed into ~~paramagnets~~ permanent

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magnets - Artificial Magnets.

- A permanent magnet does not require a continuous supply of electrical energy to maintain its magnetic field.

★ Electromagnet

- Magnetization is done by passing electric current in a coil surrounding the material is called electromagnet.

- Strength of electromagnet varies according to the flow of electric current into it.

- An electromagnetic magnet only displays magnetic properties when an electric current is applied to it.

- An electromagnet's magnetic field can be rapidly manipulated over a wide range by controlling the amount of electric current supplied to the electromagnet.

★ Magnetic Induction

- Phenomenon of changing magnetic substance to magnet.

- Two properties :
1) Attraction

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2) Repulsion

- Magnet has 2 end-poles
1) North pole
2) South pole.

★ Magnetic field

Space around a magnet where magnetic effect can be detected.

★ Magnetic flux

- Represents total number of magnetic lines of force in a magnetic field.
- Denoted by $\phi$
- Unit : Weber (Wb)

★ Magnetic Flux Density

- Flux passing per a unit area.
- $B$
- $Wb/m^2$ or Tesla (T)
- $B = \phi/A$.

★ Permeability / Absolute Permeability

- Ability of a material to pass/ conduct magnetic flux.

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- $\mu = 4\pi \times 10^{-7}$ H/m

- Relative permeability

- Permeability with resp to free space
- $\mu_r$
- $\mu_r = \mu/\mu_0$
- Relative permeability of air is 1.

★ Magnetic Field Intensity

- Force experienced in a unit north pole placed at that point.
- H
- N/Wb
- $B = \mu H$ $(\mu = \mu_0 \mu_r)$

★ Magneto motive force (mmf)

- Magnetic pressure that sets up magnetic flux in a magnetic circuit.

- Mmf = no of turns ~~in~~ of coil × current in it
- Unit AT (ampere turns).

★ Reluctance

- Opposition offered to magnetic lines of force in a magnetic circuit.

- S
- AT/Wb

$l/\mu A$

- → $l$ - length of magnetic path
- → A - cross sectional area.

- **Permeance**
- Reciprocal of reluctance
- Wb/AT

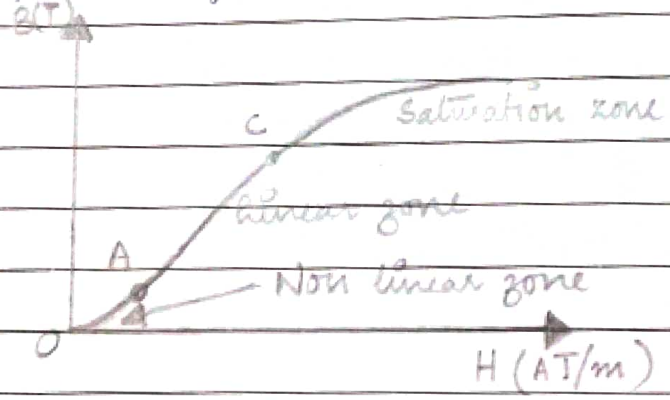
- **Electric and Magnetic Circuit - comparison**

| Electric Circuit | Magnetic Circuit |
| --- | --- |
| Path traced by the current is known as electric current. | Path traced by the magnetic flux is called as magnetic circuit. |
| EMF is the driving force in the electric circuit. The unit is volts. | MMF is the driving force in the magnetic circuit. The unit is ampere turns. |
| There is a current I in the electric circuit which is measured in amperes. | There is a flux $\phi$ in the magnetic circuit which is measured in weber. |
| The flow of electrons decides the current in conductor. | The number of magnetic lines of force decides the flux. |
| Resistance (R) oppose the flow of the current. The unit is Ohm. | Reluctance (S) is opposed by magnetic path to the flux. The unit is Aturn/wb |
| $R = \rho l/A$ Directly proportional to $l$. Inversly proportional to A. Depends on nature of material. | $S = l/\mu_0\mu_r A$ ~~S = l/~~ Directly proportional to $l$. Inversly proportional to $\mu = \mu_0\mu_r$. Inversly proportional to A. |

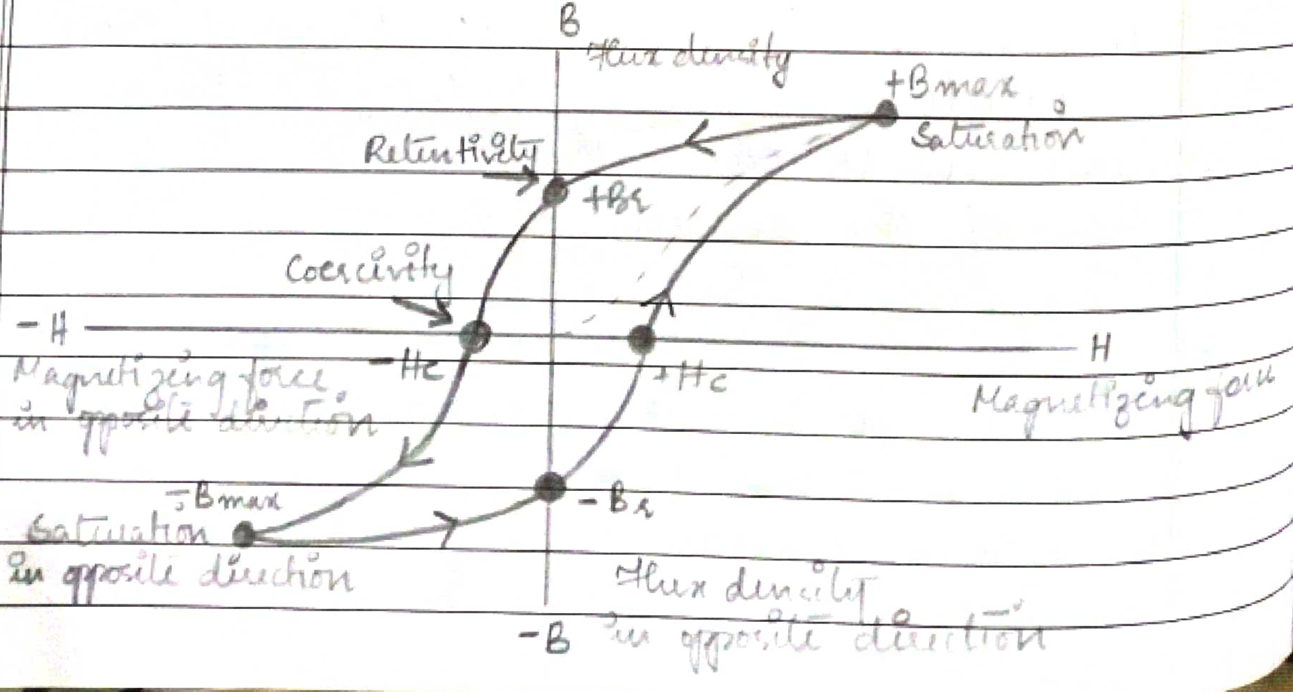
| | |
|---|---|
| • The current I = EMF / resistance. | The flux = MMF / Reluctance |
| • The current density | The flux density |
| • Kirchhoff current law (voltage) law is applicable to the electric circuit. | Kirchhoff mmf law & flux law is applicable to the magnetic flux. |

★ Magnetization Curve (BH Curve)

• Graph between magnetic flux density (B) and magnetic force (H).




★ Magnetic Hysterisis

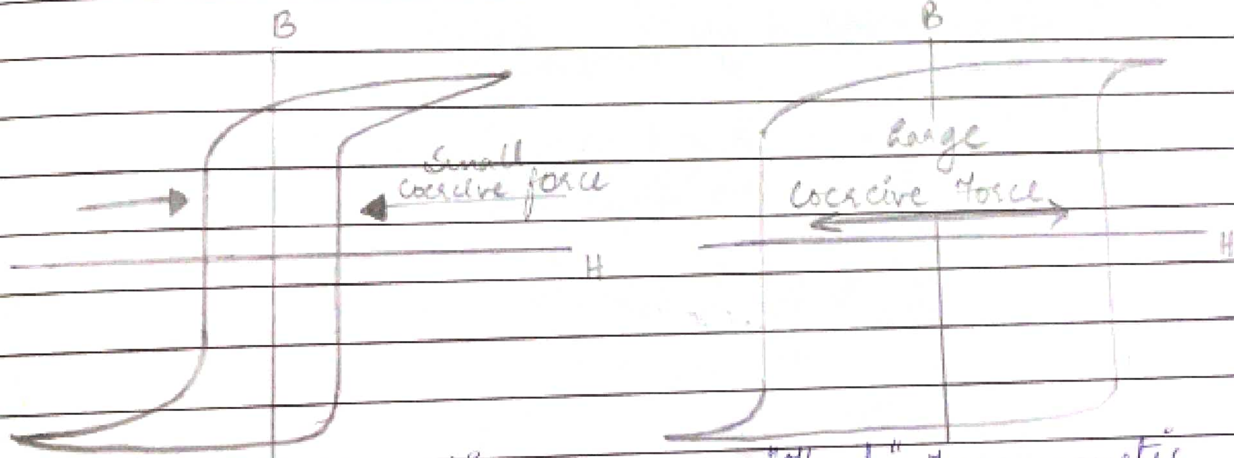


★ **Retentivity**

It is defined as the degree to which a magnetic material gains its magnetism after magnetizing force (H) is reduced to zero.

★ **Coercivity**

The amount of reverse driving field required to demagnetize it is called its coercivity.




"Soft" Ferromagnetic material

"Hard" Ferromagnetic material

★ **Magnetic Circuit**

Closed path followed by magnetic lines of force.

- $S = \frac{l}{\mu_0 \mu_r A} = \frac{l}{\mu A}$

- $l$ is the length in 'm'.
- $\mu_0$ is the permeability of vaccum, equal to $4\pi \times 10^{-7} \frac{\text{henry}}{\text{metre}}$.

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- $\mu_r$ is the relative permeability of the material
- $\mu$ is the permeability of the material ($\mu = \mu_0 \mu_r$)
- A is the cross-sectional area of the circuit in square metres.

* Composite magnetic circuit

- $S = \frac{l_1}{\mu_0 \mu_{r1} A_1} + \frac{l_2}{\mu_0 \mu_{r2} A_2} + \frac{l_g}{\mu_0 \mu_{rg} A_g}$

total MMF = flux × reluctance (S)

$$= \phi \times \left[ \frac{l_1}{\mu_0 \mu_{r1} A_1} + \frac{l_2}{\mu_0 \mu_{r2} A_2} + \frac{l_g}{\mu_0 A_g} \right] \quad \left\{ \begin{array}{c} \mu_{rg} = 1 \\ (\text{air}) \end{array} \right\}$$

Magnetic flux density,

$$B = \frac{\phi}{A}$$

$$\therefore \text{total MMF} = \frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_g l_g}{\mu_0}$$

Q1) An iron ring having cross-sectional area of $400\ mm^2$ and mean circumference of 500 mm carries a coil of 250 turns wound uniformly around it. Calculate

a) Reluctance of the ring.

b) Current required to produce a flux of 1000$\mu$Wb in the ring.

Relative permeability of iron is 400.

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Ans) 1) $S = \frac{l}{\mu_0 \mu_r A}$

$l = 500 \times 10^{-3} m$

$A = 400 \times 10^{-6} m^2$

$n = 250$

$\mu_r = 400$

$\mu_0 = 4\pi \times 10^{-7}$

$S = \frac{500 \times 10^{-3}}{4\pi \times 10^{-7} \times 400 \times 400 \times 10^{-6}}$

$= \frac{500 \times 10^{-3} \times 10^{7} \times 10^{6}}{16 \times 4 \times 10^{4} \times \pi}$

$= \frac{500}{64\pi} \times \frac{10^{10}}{10^{4}}$

$= \frac{500 \times 10^{6}}{64\pi}$

$S = 2.48 \times 10^{6}$ AT/wb

b) $I = ?$

$\phi = 1000 \mu wb = 1000 \times 10^{-6} wb$

$mmf = \phi \times S$

$NI = \phi \times S$

$I = \frac{\phi \times S}{N}$

$= \frac{\cancel{1000}^{\cancel{20}\,4} \times 10^{-6} \times 2.48 \times 10^{6}}{\cancel{250}}$

$= 4 \times 2.48$

$= 9.92 A$

Q2) A mild steel ring has a mean circumference of 500mm and a uniform cross-sectional

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area of 300 mm$^2$. Calculate the mmf required to produce a flux of 500 $\mu$wb? Assume $\mu_r$ = 1200.

Ans) $l = 500\,mm$

$= 500 \times 10^{-3}\,m$

$A = 300\,mm^2$

$= 300 \times 10^{-6}\,m$

$\mu_r = 1200$

$\mu_0 = 4\pi \times 10^{-7}$

$$S = \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{500 \times 10^{-3}}{4\pi \times 10^{-7} \times 1200 \times 300 \times 10^{-6}}$$

$$= \frac{500 \times 10^{-3} \times 10^{7} \times 10^{6}}{144 \times \pi \times 10^{4}}$$

$$= \frac{500}{144\pi}\, 10^{6}$$

$$= 1.10 \times 10^{6}\ AT/wb$$

$mmf = \phi \times S$

$= 500 \times 10^{-6} \times 1.1 \times 10^{6}$

$= 550\ AT$

23) An iron ring of mean length 60 cm has an air gap of 2mm and a winding of 300 turns. If the relative permeability of iron used in the ring is 400 when a ~~circuit~~ current of 1.5 A flows through it, find

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the flux density ?

Ans) $l = 60\,cm$

$= 60 \times 10^{-2}\,m$

$l_g = 2\,mm$

$= 2 \times 10^{-3}\,m$

$n = 300$

$\mu_r = 400$

$I = 1.5\,A$

$B = ?$

$mmf = Hl$

$mmf_1 = H_1 l_1$

$B = \mu H$

$$H = \frac{B}{\mu_0 \mu_r}$$

$$\therefore\ mmf_1 = \frac{B}{\mu_0 \mu_r} \times l_1$$

$$mmf_2 = \frac{B l_2}{\mu_0 \mu_r} \quad \{\mu_r = 1\ air\}$$

$$= \frac{B l_2}{\mu_0}$$

total $mmf = mmf_1 + mmf_2$

$$N \times I = \frac{B l_1}{\mu_0 \mu_r} + \frac{B l_2}{\mu_0}$$

$$300 \times 1.5 = \frac{B}{\mu_0} \left\{ \frac{l_1}{\mu_r} + l_2 \right\}$$

$$300 \times 1.5 \times \mu_0 = B \left\{ \frac{60 \times 10^{-2}}{400} + 2 \times 10^{-3} \right\}$$

$$450 \times 4\pi \times 10^{-7} = B \left\{ \frac{60 \times 10^{-2} + 8 \times 10^{-1}}{400} \right\}$$

$$\frac{450 \times 400 \times 4\pi \times 10^{-7}}{60 \times 10^{-2} + 8 \times 10^{-1}} = B$$

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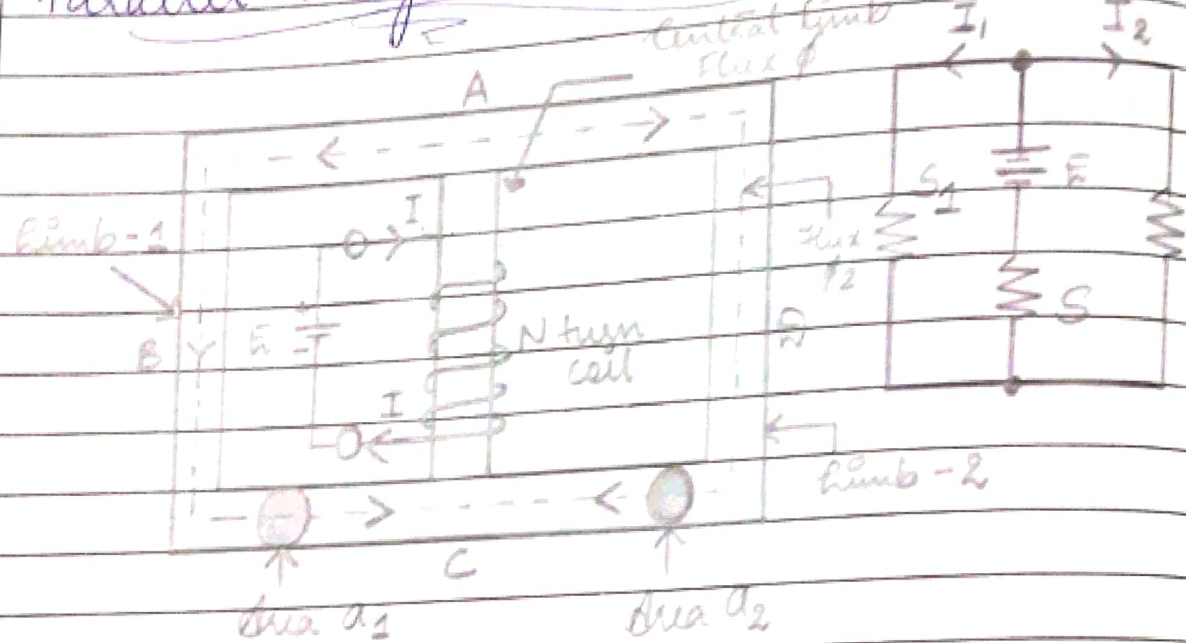
$$B = \frac{2260800 \times 10^{-7}}{1.4}$$

$$= 1614857.143 \times 10^{-7}$$

$$= 0.1615 \ T$$

## * Parallel Magnetic circuit




The total m.m.f produced by the coil of N turns is,

Total mmf = $N \times I$ (AT)

Total m.m.f can also be expressed as,

total m.m.f = Total reluctance * Total flux

m.m.f for path ABCA : F = MMF of path ABC + MMF of path AC

Thus total MMF = MMF of central limb + MMF of limb - 1 or 2

$\therefore \ S \times \phi = NI = \phi_c S_c + |\phi_1 S_1 \text{ or } \phi_2 S_2|$

The reluctances $S_1$, $S_2$, and $S_c$ are given by

$$S_1 = \frac{l_1}{\mu a_1}, \quad S_2 = \frac{l_2}{\mu a_2}, \quad S_c = \frac{l_c}{\mu a_c}$$

Assuming the cross sectional area of the three limbs to be same i.e

$$a_1 = a_2 = a_c = a.$$

the expression for $S_1$, $S_2$, $S_c$ gets modi-fied as

$$S_1 = \frac{l_1}{\mu a}, \quad S_2 = \frac{l_2}{\mu a}, \quad S_c = \frac{l_c}{\mu a}$$

Substituting these values in equations

Total MMF,

$$F = \phi_1 \times \frac{l_1}{\mu a} + \phi_c \times \frac{l_c}{\mu a}$$

$$\therefore \; F = \frac{B_1 l_1}{\mu} + \frac{B_c l_c}{\mu}$$

$$= \frac{B_1 l_1 + B_c l_c}{\mu}$$

and $F = \dfrac{B_2 l_2}{\mu} + \dfrac{B_c l_c}{\mu}$

$$= \frac{B_2 l_2 + B_c l_c}{\mu}$$

But $(B/\mu) = H$

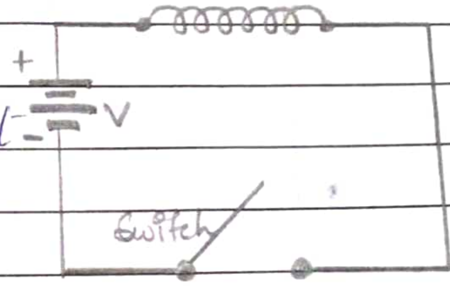
$\therefore$ For loop ABCA, $MMF(F) = H_1 l_1 + H_c l_c$

and for loop ADCA, $MMF(F) = H_2 l_2 + H_c l_c$

* Energy stored in a magnetic field

.) $e = L\,di/dt$

.) $V = iR + L\,di/dt$




. Multiplying through out by $i.dt$

$$Vi\,dt = i^2R\,dt + Li\,di$$

. Energy absorbed by the magnetic field during time $dt$

$$= Li\,di \text{ Joules}$$

. Total energy $= \int_0^I Li.\,di$

$$= L\int_0^I i\,di$$

$$= L\left[\frac{i^2}{2}\right]_0^I$$

$$= \frac{1}{2}LI_0^2$$

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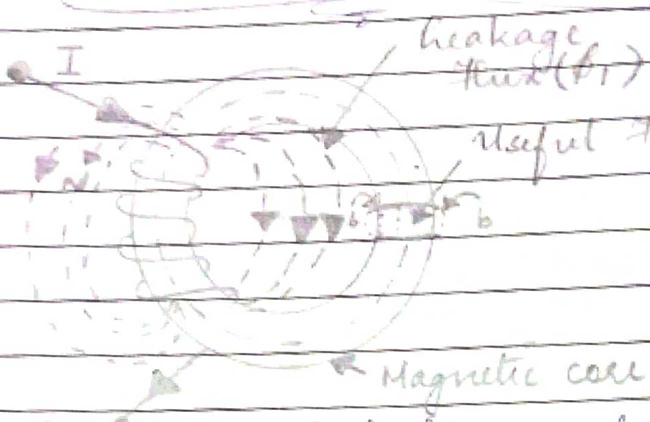
- Leakage and fringing in magnetic circuit

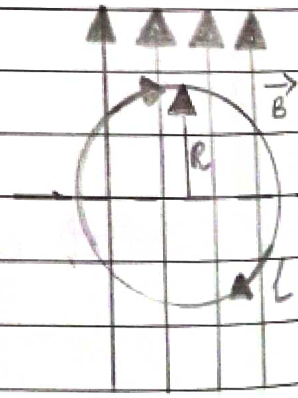



- Total flux = useful flux + leakage flux
- Leakage factor

$$\lambda = \frac{\text{total flux}}{\text{useful flux}}$$

★ Force experienced by a current carrying conductor in a magnetic field

- F = Bil (newton) ; F = Bil sinθ
  B -> ~~flux d~~ flux density
  i -> current through the conductor (A)
  l -> conductor length




Q1) A circular iron ring having cross sectional area of 20cm² and length 30cm in iron has an airgap of 2mm made by saw cut.

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Relative permeability of iron is 900. The ring is wound with a coil of 2500 turns and current in the coil is 3A. Determine air gap flux. Given leakage coefficients is 1.1.

Ans) $A = 20 cm^2$

$= 20 \times 10^{-4} m$

$l = 30 cm$

$= 30 \times 10^{-2} m$

$l_a = 2 mm$

$= 2 \times 10^{-3} m$

$\mu_r = 900$

$n = 2500$

$I = 3A$

$\lambda = 1.1$

$$\lambda = \frac{\text{total flux}}{\text{useful flux}}$$

$$1.1 = \frac{\phi_{total}}{\phi_g}$$

$$\phi_g = \frac{\phi_{total}}{1.1}$$

$$NI = \phi_{total} \times S_{total}$$

$$S_{total} = S_I + S_A$$

$$= \frac{l_I}{\mu_0 \mu_r A} + \frac{l_A}{\mu_0 A}$$

$$= \frac{30 \times 10^{-2} + 2 \times 10^{-3} \times 900}{4\pi \times 10^{-7} \times 900 \times 20 \times 10^{-4}}$$

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$$= \frac{30 \times 10^{-2} + 18 \times 10^{-1}}{4 \times 9 \times 2 \times 3.14 \times 10^{-11}}$$

$$= \frac{0.30 + 1.8}{22608 \times 10^{-11}}$$

$$S = \frac{2.1 \times 10^{-11}}{22608}$$

$$\phi_{total} = \frac{NI}{S_{total}} = \frac{2500 \times 3 \times 22608 \times 10^{-11}}{2.1}$$

$$= \frac{2500 \times 3}{9.288 \times 10^{-16}} = 8074285 71.4 \times 10^{-11}$$

$$= \frac{2500 \times 3 \times 10^{+16}}{9.288}$$

$$\phi_g = \frac{\phi_{total}}{1.1}$$

$$= \frac{8074285 71.4 \times 10^{-11}}{1.1}$$

$$= 734025974 \times 10^{-16}$$

$$\phi_g = 7.340 \times 10^{-3} \ Wb$$

★ Induced e.m.f

• Two types:

•> Dynamically induced e.m.f

•> Statically induced e.m.f

★ Dynamically Induced emf

- By moving a conductor in a uniform magnetic field and e.m.f produced in this way is known as dynamically induced e.m.f.

- Area swept by conductor = $l\,dx$

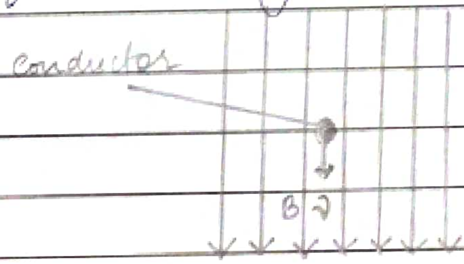
- Flux cut by the conductor = flux density × area
$= B \cdot l \cdot dx$ (Weber)

- According to Faraday's law

$e$ = rate of change of flux linkage

$= Bl\dfrac{dx}{dt}$

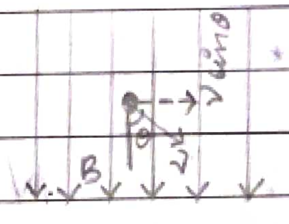
$= BlV$ (volt)




where $v = \dfrac{dx}{dt} \rightarrow$ velocity.

- Dynamically induced e.m.f = $BlV$ (volt)

$e = BlV\sin\theta$ (volt)




★ Statically Induced e.m.f

- By changing the magnetic flux

eg: transformers.

- Magnitude and direction can be changed

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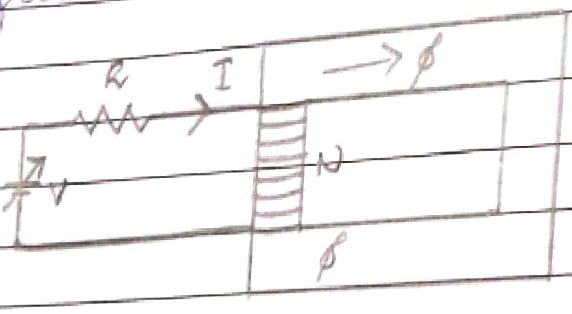
- Two types:
  - Self induced e.m.f
  - Mutually induced e.m.f

★ Self induced e.m.f

- Self-induced e.m.f is the e.m.f induced in the coil due to the change of its own flux linked with it.




- The property of coil, which opposes a change of current or flux through it, is called its self inductance, L.

★ Self inductance of the coil

- $e = -N \frac{d\phi}{dt}$ — ①

$N\phi \propto I$

$= \frac{-d(N\phi)}{dt}$

$e = \frac{-dI}{dt} \times L$ — ② {L is the self inductance}

- Unit of inductance – Henry

$L = -\frac{e}{dI/dt}$

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$$1\ \text{henry} = \frac{1\ \text{volt}}{1\ \text{ampere/second}}.$$

- 1 henry is the amount of inductance of a coil in which rate of change of current of one ampere induces an e.m.f of one volt.

- Comparing eqns ① and ②

$$N\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

integrating

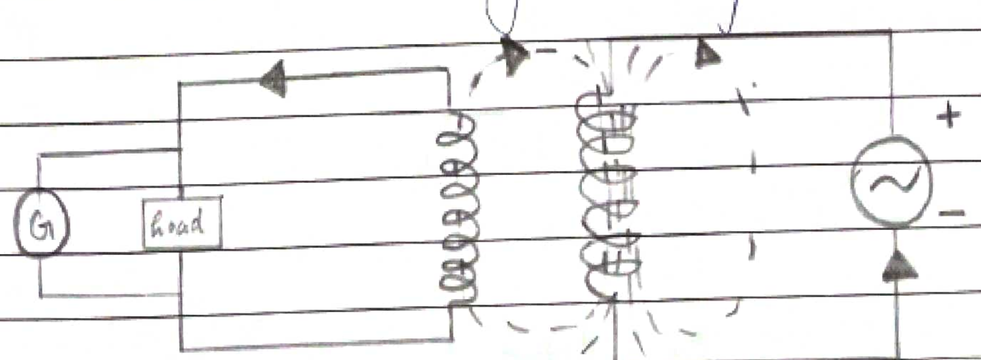
$$N\phi = LI$$

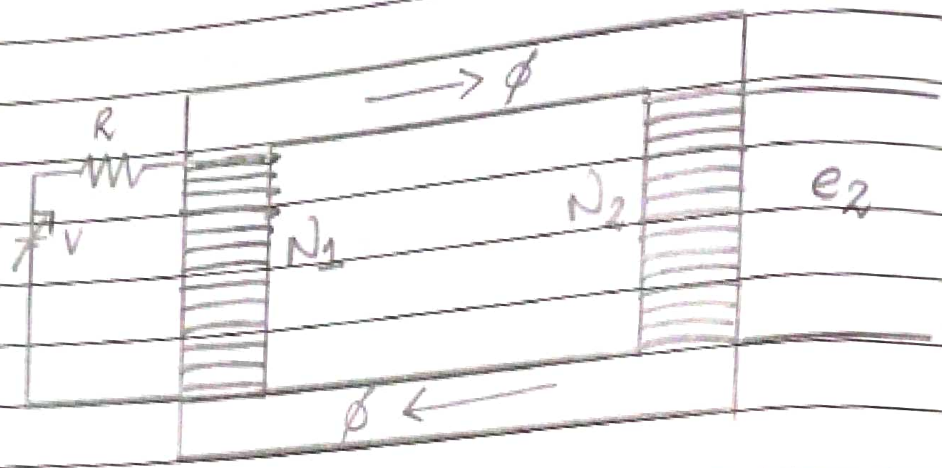
$$L = \frac{N\phi}{I}$$

- Self inductance of the coil is the flux linkage per ampere.

* Mutually induced e.m.f

- Mutual induction : Generation of induced emf in a circuit by changing the current in the neighbouring circuit.






$$e_2 = -N_2 \frac{d\phi}{dt} \quad -① $$

{-ve, it is oppos-ing the cause producing it}

But $\frac{d\phi}{dt} \propto \frac{dI_1}{dt}$

$$e_2 \propto -\frac{dI_1}{dt}$$

$$e_2 = -M\frac{dI_1}{dt} \quad -② $$

Equating ① and ②

$$N_2 \frac{d\phi}{dt} = M\frac{dI_1}{dt}$$

integrating,

$$N_2\phi = MI_1$$

$$M = \frac{N_2\phi}{I_1}$$

Similarly,

$$M = \frac{N_1\phi}{I_2}$$

* Coefficient of coupling

* $\phi_2 = k_1 \phi_1$

$$M = \frac{N_2 \phi_2}{I_1} = \frac{N_2 k_1 \phi_1}{I_1} \quad - (1)$$

$\phi_2$ produced in second coil.

$$M = \frac{N_1 \phi_1}{I_2} = \frac{N_1 k_2 \phi_2}{I_2} \quad - (2)$$

Multiplying (1) & (2)

$$M^2 = \frac{N_2 k_1 \phi_1 \times N_1 k_2 \phi_2}{I_1 \times I_2}$$

$$M^2 = k_1 k_2 L_1 L_2$$

$\{k = \sqrt{k_1 k_2}\}$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

{K is called coefficient of coupling.}

* Self Inductance of a solenoid

$l \rightarrow$ length of the solenoid
$N \rightarrow$ number of turns
$I \rightarrow$ currents through the solenoid
$A \rightarrow$ Area of cross-section of the solenoid
$L \rightarrow$ Self inductance of the solenoid.

$$\text{Flux } (\phi) = \frac{\text{MMF}}{\text{Reluctance}}$$

$$\phi = \frac{N_2 I}{l / \mu_0 \mu_r A} \quad - (1)$$

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But $L = \frac{N\phi}{I}$

$\phi = \frac{LI}{N}$ — (2)

Comparing equations (1) and (2)

$$\frac{LI}{N} = \frac{NI}{l/A(\mu_0 \mu_r)}$$

$$L = \frac{N^2 A(\mu_0 \mu_r)}{l}$$

But reluctance $(S) = \frac{l}{A(\mu_0 \mu_r)}$

$$\therefore L = \frac{N^2}{S}$$

Q) Derive the expression for effective inductance when 2 coils are connected in

1) Series
2) parallel.

Ans) (i) Consider induced emf across each conductor

a) aiding

$$V = L' \frac{dI}{dt} \quad — (1)$$

$$V_1 = L_1 \frac{dI}{dt} + M \frac{dI}{dt} \quad — (2)$$

$$V_2 = L_2 \frac{dI}{dt} + M \frac{dI}{dt} \quad — (3)$$

total voltage $= V_1 + V_2$

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$$② + ③ = L_1 \frac{dI}{dt} + M\frac{dI}{dt} + L_2 \frac{dI}{dt} + M\frac{dI}{dt}$$

$$V = (L_1 + L_2 + 2M)\frac{dI}{dt}$$

$\therefore$ from ①

$$L' = L_1 + L_2 + 2M$$

Effective inductance when 2 coils are connected in series.

b) opposing.

$$V = L''\frac{dI}{dt} \quad -①$$

$$V_1 = L_1\frac{dI}{dt} - M\frac{dI}{dt} \quad -②$$

$$V_2 = L_2\frac{dI}{dt} - M\frac{dI}{dt} \quad -③$$

$$② + ③ = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} - 2M\frac{dI}{dt}$$

$$= (L_1 + L_2 - 2M)\frac{dI}{dt}$$

$\therefore$ from ①

$$L'' = L_1 + L_2 - 2M$$

Effective inductance when 2 coils are connected in series.

(ii) Parallel conn<sup></sup>ection

$$V = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} \quad -① $$

$$V = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} \quad -② $$

from ①

$$\frac{V - M\frac{dI_2}{dt}}{L_1} = \frac{dI_1}{dt} \quad -③ $$

Sub ③ in ②

$$V = L_2 \frac{dI_2}{dt} + M \left( \frac{V - M\frac{dI_2}{dt}}{L_1} \right)$$

$$V = \frac{L_1 L_2 \frac{dI_2}{dt} + MV - M^2 \frac{dI_2}{dt}}{L_1}$$

$$VL_1 - MV = (L_1 L_2 - M^2) \frac{dI_2}{dt}$$

$$\frac{dI_2}{dt} = \frac{V(L_1 - M)}{L_1 L_2 - M^2} \quad -④ $$

Sub ④ in ③

$$\frac{dI_1}{dt} = \frac{V - M \times \frac{V(L_1 - M)}{L_1 L_2 - M^2}}{L_1}$$

$$= \frac{V(L_1 L_2 - M^2) - MVL_1 + M^2 V}{L_1 (L_1 L_2 - M^2)}$$

$$= \frac{VL_1 L_2 - \cancel{VM^2} - MVL_1 + \cancel{M^2 V}}{L_1 (L_1 L_2 - M^2)}$$

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$$= \frac{V \cancel{L_1}(L_2 - M)}{\cancel{L_1}(L_1L_2 - M^2)}$$

$$\frac{dI_1}{dt} = \frac{V(L_2 - M)}{L_1L_2 - M^2}$$

$$\therefore \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$= \frac{V(L_2 - M)}{L_1L_2 - M^2} + \frac{V(L_1 - M)}{L_1L_2 - M^2}$$

$$\frac{dI}{dt} = \frac{V(L_1 + L_2 - 2M)}{L_1L_2 - M^2}$$

$$V = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} \frac{dI}{dt}$$

$$V = L_p \frac{dI}{dt}$$

$$L_p = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

Effective inductance when 2 coils are connected in parallel.

Q) Two coupled coils connected in series have an equivalent inductance of 0.725H when aiding and 0.425 H when opposing. Find self and mutual inductance when K = 0.42.

Ans) $L' = 0.725H$ ; $L'' = 0.425\ H$

$K = 0.42$

$$\frac{M}{\sqrt{h_1 h_2}} = 0.42$$

$$0.725 = h_1 + h_2 + 2M \quad -(1)$$
$$0.425 = h_1 + h_2 - 2M \quad -(2)$$

(1) + (2)

$$0.575 = h_1 + h_2 \quad - (3)$$

(2) − (1)

$$0.03 = 2M$$
$$M = 0.075$$

∴ Sub $M = 0.075$ and (3) in $K$

$$0.42 = \frac{0.075}{\sqrt{(0.575 - h_2) h_2}}$$

$$h_2^2 - 0.575 h_2 + 0.03188 = 0$$

$$D = b^2 - 4ac = \sqrt{(-0.575)^2 + 4(1)(0.03188)}$$
$$= \sqrt{0.203055} = 0.4506$$

$$h_2 = \frac{-b \pm D}{2a}$$

$$h_2 = \frac{-(-0.575) + 0.4506}{2} \; ; \; h_2 = \frac{0.575 - 0.4506}{2}$$

∴ when $h_2 = 0.5126$ ; $h_1 = 0.0624$

and when $h_2 = 0.0624$ ; $h_1 = 0.5126$

$$M = 0.075$$

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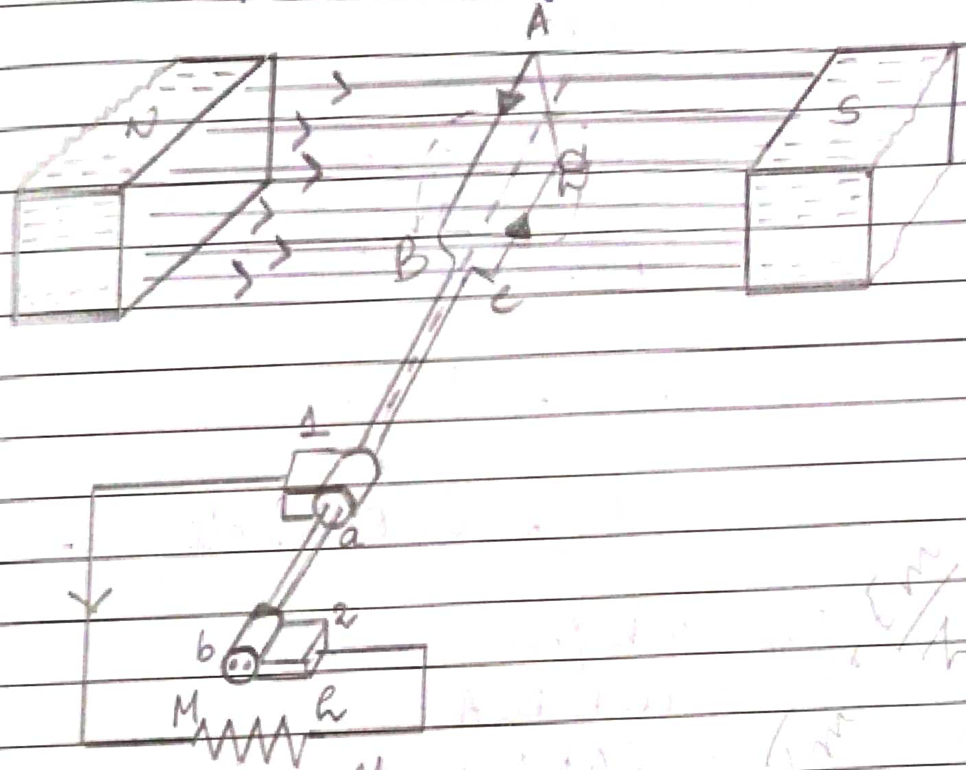
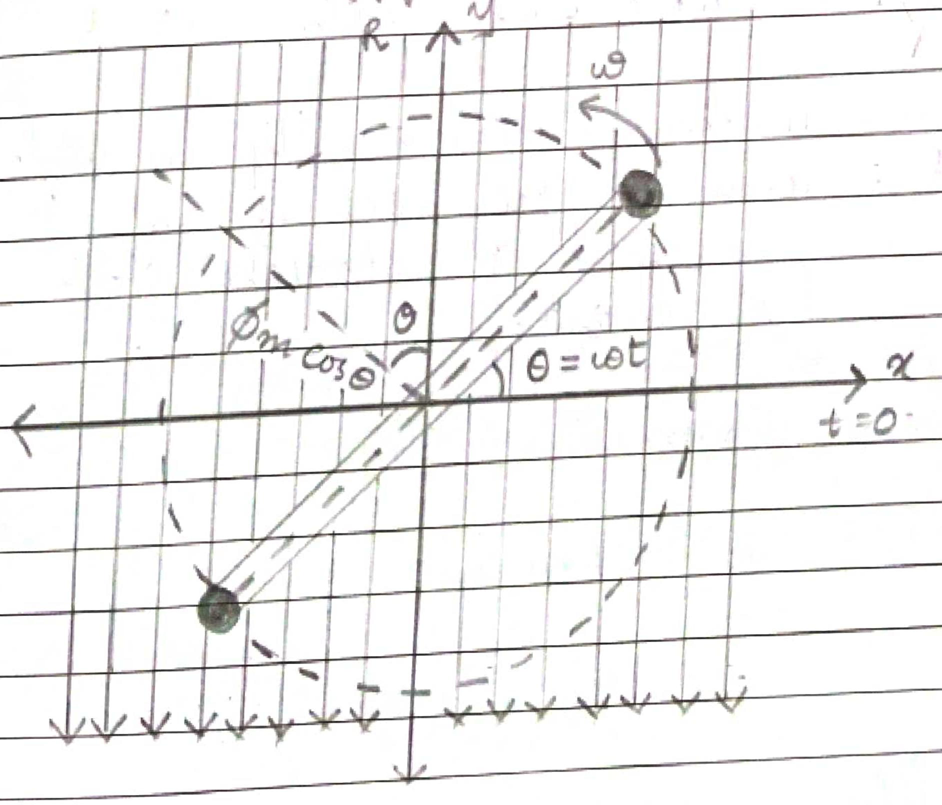
# Alternating E.m.f.

## Production of alternating emf




$\theta = \omega t$ — ①

$\phi = \phi_m \cos \omega t$

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$N\phi = N\phi_m \cos\theta$ {from ①}

$$e = -\frac{d(N\phi)}{dt}$$

$$= -\frac{d(N\phi_m \cos\theta)}{dt}$$

$$= -N\frac{d(\phi_m \cos\theta)}{dt}$$

$$= -N\phi_m\,\omega(-\sin\omega t) \text{ volt}$$

$$= N\omega\phi_m \sin\omega t$$

$$e = \omega N\phi_m \sin\theta \text{ volt.} \quad — ②$$

$E_m = \omega N\phi_m$

$= \omega N B_m A$

$= 2\pi f N B_m A$ volt $— ③$ $\left\{\begin{matrix}\omega = 2\pi f \\ \phi_m = B_m A\end{matrix}\right\}$

$B_m$ : maximum flux density in $Wb/m^2$
A : area of the coil in $m^2$
f : frequency of rotation of the coil in (rev/second.)

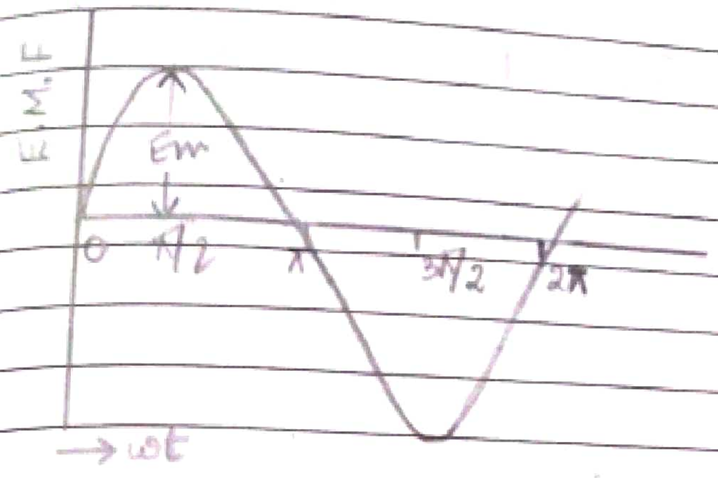
∴ from ② & ③

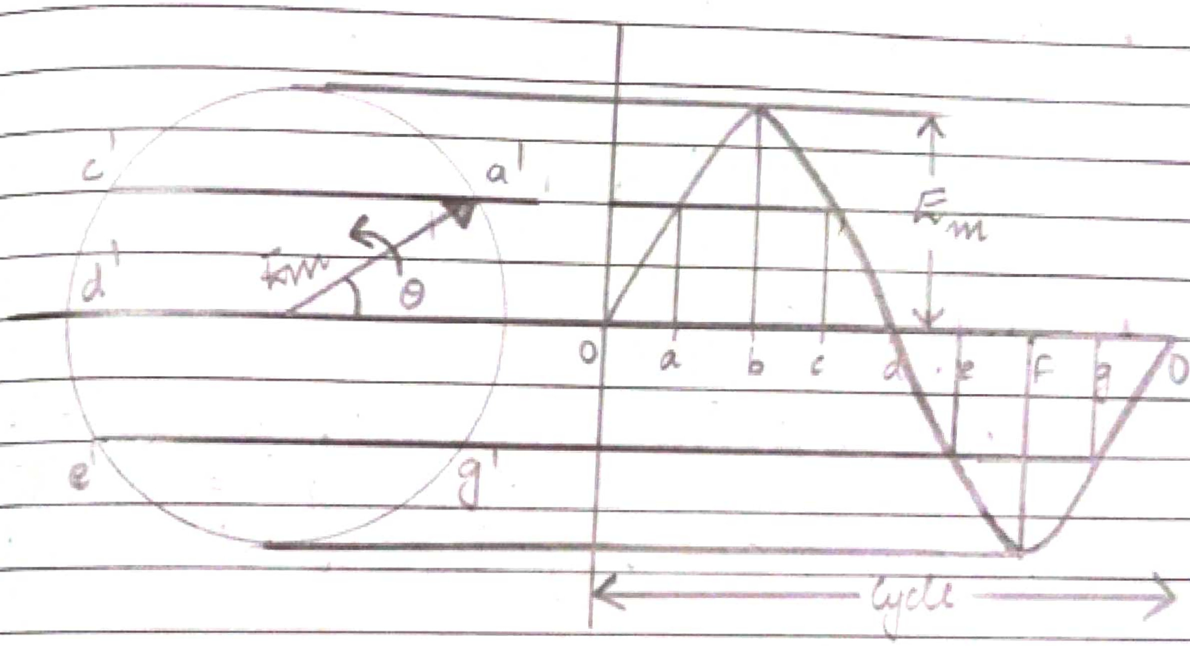
$$e = E_m \sin\theta$$

$$e = E_m \sin\omega t \quad \{\text{from ①}\}$$

Illy,

$$i = I_m \sin\omega t$$ $\left\{I_m = \frac{E_m}{R}\right\}$




- Cycle

- One complete set of positive and negative values of alternating quantity is known as cycle.

- One complete cycle is said to spread over $360^\circ$ or $2\pi$

- Time period

The time taken by an alternating

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quantity to complete one cycle is called its time period. (T)

eg: 50 Hz alternating current has a time period of $\frac{1}{50}$ seconds.

* Frequency

• The number of cycles/second is known as frequency.

• unit hertz ; $f = \frac{1}{T}$

* Amplitude

The maximum value, positive or negative, of an alternating quantity is known as amplitude.

* Instantaneous value

It is the value at a particular instant.

* Average Value

It is the arithmetic mean of the ordinates at equal interval over a half cycle of a wave.

2 Methods :

1) Mid-ordinate method : graphical method
2) Analytical method.

Mid-ordinate method.

Analytical method

$$avg = \frac{Area}{\pi} = \frac{\int_0^{\pi} i \, d\omega t}{\pi} \qquad i = I_m \sin \omega t$$

RMS Value (Root mean square value)

RMS Value : That value of DC current which when flows through a given conductor produces same amount of heat as that produced by the alternating current passing through the same conductor for the same time.

$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{\int_0^{\pi} i^2 dt}{\pi}}$$

RMS value of sinewave $= \frac{I_m}{\sqrt{2}}$

Average value of sine wave = $\frac{2I_m}{\pi}$

★ **Form Factor**

Form factor = $\frac{\text{RMS value}}{\text{Avg value}}$

★ **Peak factor**

Peak factor = $\frac{\text{Max. value}}{\text{RMS value}}$

• Form factor of sine wave = $\frac{I_m/\sqrt{2}}{2I_m/\pi}$

$$= \frac{\pi}{2\sqrt{2}}$$

$$= 1.11$$

• Peak factor of sine wave = $\frac{I_m}{I_m/\sqrt{2}}$

$$= \frac{1}{\sqrt{2}}$$